

Electromagnetic Theory Teaching and Synthesis Guide

Electromagnetic Theory Course — Teaching, Synthesis and Worked Examples — Document 19 of 23

How to turn the preceding mathematics and electromagnetics into a coherent degree-level subject that you can explain, derive, demonstrate and assess.

1. The whole course as one connected story

Electromagnetics is easiest to teach when it is presented as a **chain of ideas**, not as a collection of formulas. The chain is
sources → fields → Maxwell → waves → boundaries → guided/radiated fields → numerical solution.

Charge and current are the sources. The four field quantities **E, D, B, H** describe the electromagnetic state. Maxwell's equations tell us locally and globally how sources and fields are related. Constitutive laws describe the material. Boundary conditions connect one region to another. The wave equation, transmission-line equations and waveguide modes are consequences of that framework, not independent inventions.

A student should therefore leave the course able to answer three questions for almost every equation: **what does it describe physically, where does it come from, and what assumption was used?**

A good teaching habit is to move repeatedly through four representations:

1. **picture** - field lines, surfaces, loops, wave fronts, current distributions;
2. **words** - the physical statement;
3. **equation** - the compact mathematical statement;
4. **engineering consequence** - what it changes in an antenna, waveguide, link or solver.

Do not begin with algebra when a picture can establish the idea first. The algebra should then make the picture quantitative.

Teaching objective. By the end of this guide you should be able to organise a degree-level EM module, select derivations worth teaching in full, identify misconceptions early, construct meaningful worked problems, and connect analytical field theory to computational electromagnetics.

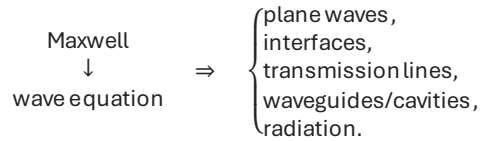
2. Dependency map: what must be secure before what

The mathematical dependencies are unusually important in EM. If a learner is weak in one layer, difficulty often appears several chapters later.

A useful dependency ladder is:

algebra $\rightarrow$ complex numbers $\rightarrow$ vectors $\rightarrow$ multivariable calculus $\rightarrow$ ∇ operators $\rightarrow$ Maxwell.

From Maxwell the subject branches:



Computational EM adds another branch:

PDE/integral equation $\rightarrow$ discretisation $\rightarrow$ matrix system $\rightarrow$ numerical field.

Before teaching Maxwell's differential equations, check that students understand partial derivatives and the distinction between a scalar and vector field. Before teaching the Poynting vector, check the cross product. Before teaching wave impedance, check phasors and complex ratios. Before teaching MoM, check Green's functions and matrix linear systems.

A useful diagnostic question is not "can you reproduce the formula?" but "what mathematical operation is being performed, and what physical quantity comes out?" For example, $\nabla \cdot \mathbf{D}$ produces a scalar source density, whereas $\nabla \times \mathbf{E}$ produces a vector describing local circulation.

3. Derivations worth teaching in full

Not every formula deserves a classroom derivation. Some should be accepted as definitions or quoted results; others are central because the derivation teaches the structure of the subject.

The following are worth deriving carefully:

- Gauss's law in differential form from its integral form using the divergence theorem;
- Faraday's and Ampere-Maxwell laws in differential form using Stokes' theorem;
- charge continuity from the divergence of Ampere-Maxwell;
- the electromagnetic wave equation from the curl equations;
- the plane-wave relationship between $\mathbf{E}$, $\mathbf{H}$ and propagation direction;
- intrinsic impedance and time-average Poynting power;
- reflection/transmission at a plane boundary;
- the transmission-line reflection coefficient and standing-wave ratio;
- rectangular-waveguide cutoff and TE/TM modal structure;
- the origin of a Green's-function integral equation;
- the MoM reduction to $\mathbf{ZI} = \mathbf{V}$.

For each derivation, identify the assumptions in a margin note. For the lossless wave equation, for example, assumptions include homogeneous ϵ, μ , source-free region and linear isotropic material. If the student cannot state those assumptions, they do not yet own the derivation.

A strong lecture ends a derivation by **reading the final equation physically**. For example, the wave equation says that spatial curvature of the field is tied to its second time derivative; the result admits disturbances that travel at $1/\sqrt{\mu\epsilon}$.

4. How to explain fields without turning them into pictures only

Field diagrams are essential, but they must not create the false impression that a field is literally made of drawn lines. A field is a function that assigns a quantity to every point in a region.

For a vector electric field,

$$\mathbf{E}(x, y, z) = E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}} + E_z \hat{\mathbf{z}}.$$

A field-line drawing samples this information visually. The line tangent gives local direction; line density is sometimes used qualitatively for magnitude, but the drawing is not the field itself.

Teach $\mathbf{E}$ and $\mathbf{D}$ together but distinguish their roles. Likewise distinguish $\mathbf{B}$ and $\mathbf{H}$. In linear isotropic media,

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H},$$

but the pairs are not interchangeable concepts. $\mathbf{D}$ and $\mathbf{H}$ are particularly useful when separating free sources from material response.

For vector calculus, use physical verbs:

- ∇V : **climbs** a scalar field;
- $\nabla \cdot \mathbf{F}$: measures **outflow/source strength**;
- $\nabla \times \mathbf{F}$: measures **circulation/rotation**.

Then immediately connect them to EM. Charge produces divergence of $\mathbf{D}$. Changing magnetic flux produces curl of $\mathbf{E}$. These are far easier to retain than coordinate expansions memorised in isolation.

5. Teaching Maxwell’s equations as a system

Present all four Maxwell equations together repeatedly. Students often learn them individually and fail to see the system.

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho_v, & \nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, & \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}. \end{aligned}$$

A useful teaching table is:

Equation	Local meaning	Global meaning
Gauss electric	charge is a source of electric flux	net electric flux counts enclosed free charge
Gauss magnetic	no magnetic monopole source term	magnetic flux through a closed surface sums to zero
Faraday	changing B produces circulating E	changing magnetic flux induces emf
Ampere-Maxwell	current and changing D produce circulating H	conduction + displacement current drive magnetic circulation

Then demonstrate feedback: a changing $\mathbf{E}$ produces $\mathbf{H}$, and a changing $\mathbf{H}$ produces $\mathbf{E}$. In a source-free region, that coupling sustains a propagating wave.

For phasor-domain teaching, explicitly announce the convention $e^{j\omega t}$. Then

$$\frac{\partial}{\partial t} \rightarrow j\omega.$$

Many apparent sign disagreements between books come from different time conventions rather than different physics.

6. Common misconceptions to diagnose early

A degree-level teacher should expect predictable misconceptions and design questions that expose them.

“B is the magnetic field and H is another name for it.” Correct this by emphasising $\mathbf{B} = \mu\mathbf{H}$ and discussing materials and boundary conditions.

“Power is $\mathbf{E} \times \mathbf{B}$.” In ordinary macroscopic EM the Poynting vector is

$$\mathbf{S} = \mathbf{E} \times \mathbf{H},$$

or equivalently $(1/\mu)\mathbf{E} \times \mathbf{B}$ in a simple linear isotropic medium.

“A waveguide mode is a ray bouncing down a pipe.” Ray pictures can aid intuition but modes are eigenfunctions satisfying Maxwell plus conducting boundaries.

“Phase velocity is the speed of information.” In a waveguide v_p may exceed c ; group velocity carries modulation/energy in the ideal nondispersive case, with $v_p v_g = c^2$ for an air-filled hollow guide.

“A solver output is true because it looks smooth.” Require convergence, energy balance, reciprocity or comparison against a known case.

“Each OFDM subcarrier seeing flat fading means the whole channel is flat.” No: each narrow subcarrier may be approximately flat while H_k varies substantially across the occupied bandwidth.

Misconception questions are excellent formative assessment because they reveal structure, not just arithmetic ability.

7. Teaching numerical electromagnetics

Students often see MoM, FEM and FDTD as unrelated software technologies. Teach them as different numerical representations of the same field laws.

For a driven frequency-domain problem:

$$\mathbf{Ax} = \mathbf{b}.$$

The vector $\mathbf{x}$ contains unknown field/current coefficients. The matrix $\mathbf{A}$ represents how the chosen basis/testing functions interact through the governing operator.

For an eigenmode problem:

$$\mathbf{Ke} = \omega^2 \mathbf{Me}.$$

The eigenvector is a field shape; the eigenvalue determines a resonance/cutoff quantity.

Explain the methods through their natural viewpoints:

- **MoM:** source-to-source interaction through an integral kernel/Green's function;
- **FEM:** weak form on local volume elements;
- **FDTD:** direct time stepping of Maxwell's curl equations;
- **eigenmode:** natural source-free field patterns.

Every numerical lecture should include one analytical benchmark. Examples include a parallel-plate field, rectangular TE₁₀, a dipole trend, or scattering from a simple cylinder. Numerical literacy means knowing when not to trust the solver.

8. Designing worked problems and assessment

Good EM problems should test interpretation as well as calculation. A useful progression has four levels.

Level 1 - identify. What is the correct law/operator/coordinate system?

Level 2 - calculate. Substitute correctly with units and conventions.

Level 3 - derive. Start from a more fundamental equation and produce the result.

Level 4 - interpret/design. Explain the engineering consequence or choose between approaches.

For example, a waveguide question might ask students to:

1. calculate f_c for TE₁₀;
2. compute β , λ_g , v_p and v_g ;
3. sketch the transverse electric field;
4. explain why $v_p > c$ does not violate relativity;
5. predict what happens as $f \rightarrow f_c^+$.

Marking should reward assumptions and units. A technically correct numeric answer with the wrong physical interpretation should not receive full credit.

For computational tasks, ask for a convergence plot, not only a field picture. Require a statement of mesh/basis resolution and a physical cross-check.

9. Oral teaching, viva questions and demonstrations

Oral explanation is a powerful test of deep understanding. Useful questions include:

1. Why does a divergence produce a scalar?
2. Why is the flux through a closed surface sensitive to sources inside?
3. What is displacement current physically doing in Maxwell's equations?
4. How can $v_p > c$ in a waveguide while causality is preserved?
5. Why is a 2D free-space Green's function a Hankel function rather than e^{-jkr}/r ?
6. Why is a MoM matrix normally dense?
7. Why can an FDTD solution become unstable?
8. Why does an eigenvector have arbitrary scale?
9. What does antenna reciprocity let us reuse between transmit and receive?
10. How would you prove that a numerical result is converged?

Demonstrations should be chosen to make invisible quantities visible: iron-filings-like field analogies, a slotted-line/standing-wave animation, rotating phasors, a rectangular-waveguide field animation, and mesh-refinement comparisons.

A useful rule for teaching is **one new abstraction at a time**. If the physics is new, keep the geometry simple. If the geometry is complicated, use familiar physics. This prevents algebra, coordinates and physical interpretation from all becoming difficult simultaneously.

10. A 12-week degree-level teaching plan and capstone

A practical 12-week sequence could be:

Week	Theme	Core documents
1	notation, vectors, complex numbers	1-3
2	multivariable calculus, grad/div/curl	4, 6
3	fields, materials, sources	5
4	Maxwell integral and differential forms	7
5	phasors and wave equation	8-9
6	plane waves, polarization, power	10
7	boundaries and reflection	11
8	transmission lines	12
9	waveguides and cavities	13
10	numerical EM overview	14
11	Green's functions, MoM, FEM/FDTD	15-17
12	radiation, antennas, synthesis	18-19

A strong **capstone** is to analyse one structure analytically and numerically. For example, use a rectangular waveguide: derive TE₁₀, calculate cutoff and guide wavelength, compute power flow, then compare with an eigenmode model. Or use a 2D PEC cylinder: formulate the scattering integral equation, discretise it with MoM, solve for current, and calculate a far-field pattern.

The standard for “ready to teach” is not perfect memory. It is the ability to reconstruct the argument from fundamentals, state assumptions, draw the geometry, use correct units, explain limiting cases and answer “why?” without hiding behind a formula.

Final self-test: choose any equation from Documents 1-18. Explain every symbol, state its assumptions, derive or motivate it, give one physical example, and describe how a numerical solver would represent it. If you can do that reliably, you have moved from learning EM to being able to teach it.