

Maxwell's Equations

Electromagnetic Theory Course — Maxwell and Wave Theory — Document 7 of 23

1. Maxwell's equations: the framework of classical electromagnetics

Maxwell's equations are the compact set of laws from which most classical electromagnetic behaviour can be derived. They connect **electric charge**, **electric field**, **magnetic field**, **electric current**, and the way electric and magnetic fields generate one another when they vary with time.

In macroscopic SI form, the four equations are

$$\begin{aligned}\nabla \cdot \mathbf{D} &= \rho_f, \\ \nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\ \nabla \times \mathbf{H} &= \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}.\end{aligned}$$

The symbols are not interchangeable. The field quantities are

Quantity	Meaning	SI unit
$\mathbf{E}$	electric field strength	V/m
$\mathbf{D}$	electric flux density	C/m ²
$\mathbf{H}$	magnetic field strength	A/m
$\mathbf{B}$	magnetic flux density	T

and the sources are

Quantity	Meaning	SI unit
ρ_f	free volume charge density	C/m ³
$\mathbf{J}_f$	free current density	A/m ²

For a simple linear, isotropic material,

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}, \quad \mathbf{J} = \sigma \mathbf{E}.$$

These are **constitutive relations**. Maxwell's equations describe the field laws; the constitutive relations describe how the material responds.

There are two complementary forms of Maxwell's equations. The **differential form** describes what happens in an infinitesimal neighbourhood around every point. The **integral form** describes accumulated field behaviour over a finite line, surface or volume. Neither is more fundamental for ordinary engineering use; each is convenient for different problems.

A useful classification is:

- the two **divergence equations** describe sources and flux;
- the two **curl equations** describe circulation and electromagnetic induction.

The four laws are strongly coupled. A time-varying magnetic field can create a circulating electric field; a time-varying electric flux can create a circulating magnetic field. That mutual coupling is what ultimately permits electromagnetic waves to propagate through space.

By the end of this document you should be able to state all four equations in integral and differential form, explain every term physically, derive the differential forms using Gauss's and Stokes' theorems, explain displacement current and charge conservation, and use Maxwell's equations to reason about boundaries and wave propagation.

Core mental model: Gauss's laws tell you how field flux relates to sources. Faraday's and Ampère-Maxwell laws tell you how changing or conducting sources produce circulating fields.

2. Gauss's law for electricity

Gauss's law for electricity relates electric flux to electric charge. In differential form,

$$\nabla \cdot \mathbf{D} = \rho_f.$$

Read this as: **the local divergence of electric flux density equals the local free-charge density**. A positive charge is a source of outward electric flux; a negative charge acts as a sink.

The corresponding integral form is

$$\oiint_S \mathbf{D} \cdot d\mathbf{S} = Q_{f,\text{enc}}$$

where S is a closed surface and $Q_{f,\text{enc}}$ is the total free charge enclosed by it. Since

$$Q_{f,\text{enc}} = \iiint_V \rho_f dV,$$

we can write

$$\oiint_S \mathbf{D} \cdot d\mathbf{S} = \iiint_V \rho_f dV.$$

Using the divergence theorem,

$$\oiint_S \mathbf{D} \cdot d\mathbf{S} = \iiint_V \nabla \cdot \mathbf{D} dV,$$

so

$$\iiint_V (\nabla \cdot \mathbf{D} - \rho_f) dV = 0.$$

Because this must hold for any arbitrary volume,

$$\boxed{\nabla \cdot \mathbf{D} = \rho_f}.$$

Example: point charge

For a point charge Q in free space, spherical symmetry tells us that $\mathbf{D}$ must point radially and have constant magnitude over a sphere of radius r :

$$\mathbf{D} = D_r \hat{\mathbf{r}}.$$

Gauss's law gives

$$\oiint_S \mathbf{D} \cdot d\mathbf{S} = D_r (4\pi r^2) = Q,$$

hence

$$\mathbf{D} = \frac{Q}{4\pi r^2} \hat{\mathbf{r}}.$$

In free space, $\mathbf{D} = \epsilon_0 \mathbf{E}$, giving Coulomb's field:

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}}.$$

This is an important lesson: Gauss's law is always true, but it becomes especially powerful when symmetry makes the flux integral easy.

What zero divergence does and does not mean

If

$$\nabla \cdot \mathbf{D} = 0,$$

there is no **net local source of electric flux** at that point. It does not imply that $\mathbf{D} = 0$. A nonzero uniform field has zero divergence because as much flux enters a tiny volume as leaves it.

Teaching point: divergence answers a source/sink question, not a field-magnitude question.

3. Gauss's law for magnetism

Maxwell's second divergence equation is

$$\nabla \cdot \mathbf{B} = 0.$$

Its integral form is

$$\oiint_S \mathbf{B} \cdot d\mathbf{S} = 0.$$

This says that the net magnetic flux through any closed surface is zero. In ordinary classical electromagnetics there are no isolated magnetic charges analogous to electric charges. Magnetic field lines therefore do not begin or end on magnetic monopoles; they form continuous loops.

Consider a bar magnet. We often draw magnetic field lines leaving the north pole and entering the south pole outside the magnet. If we stopped there, it might look as though the north pole were a magnetic source and the south pole a magnetic sink. But inside the magnet the field continues from south to north, completing the loop. A closed surface around the whole magnet therefore has as much magnetic flux entering as leaving.

Differential interpretation

The divergence equation

$$\nabla \cdot \mathbf{B} = 0$$

means that an infinitesimal volume cannot contain a net source or sink of magnetic flux. This is a very useful physical constraint in both analytical and numerical EM.

For example, suppose a proposed field were

$$\mathbf{B} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}.$$

Then

$$\nabla \cdot \mathbf{B} = 1 + 1 + 1 = 3,$$

so this field cannot represent a physically permissible magnetic flux density throughout a source-free classical region.

By contrast,

$$\mathbf{B} = -y\hat{\mathbf{x}} + x\hat{\mathbf{y}}$$

has

$$\nabla \cdot \mathbf{B} = 0.$$

It is compatible with the no-monopole constraint, although additional Maxwell equations would still need to be checked before declaring it a complete physical solution.

Why solvers care about this equation

Numerical EM methods approximate continuous fields using discrete unknowns. If the discretisation is poorly chosen, it may produce nonphysical numerical modes with spurious magnetic divergence. Good formulations preserve or enforce the structure of Maxwell's equations as closely as practical.

In finite-difference and finite-element methods, the relationship between discrete curl and divergence operators is especially important. Since mathematically

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0,$$

well-designed discrete operators try to retain the corresponding identity. This helps prevent unphysical accumulation of magnetic flux.

A useful comparison

Electric field flux can terminate on charge:

$$\nabla \cdot \mathbf{D} = \rho_f.$$

Magnetic flux cannot terminate:

$$\nabla \cdot \mathbf{B} = 0.$$

That difference is one of the clearest structural asymmetries in the classical Maxwell system.

Teaching point: $\nabla \cdot \mathbf{B} = 0$ does not say that magnetic fields are weak or absent. It says that magnetic flux has no isolated beginning or end.

4. Faraday's law: changing magnetic flux creates electric circulation

Faraday's law is

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Its integral form is

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint_S \mathbf{B} \cdot d\mathbf{S}.$$

The left side is the circulation of the electric field around a closed contour C . The right side is the negative time rate of change of magnetic flux through a surface bounded by that contour.

The minus sign encodes **Lenz's law**: the induced electric effect acts in a sense that opposes the change in magnetic flux that produces it.

From field law to induced voltage

The electromotive force around a closed loop is

$$\mathcal{E} = \oint_C \mathbf{E} \cdot d\mathbf{l}.$$

Therefore Faraday's law gives

$$\mathcal{E} = -\frac{d\Phi_B}{dt},$$

where

$$\Phi_B = \iint_S \mathbf{B} \cdot d\mathbf{S}.$$

If a coil has N identical turns linking the same flux,

$$V_{\text{ind}} = -N \frac{d\Phi_B}{dt}.$$

This is the familiar transformer/generator law, but Maxwell's equation is more general: it says the induced electric field exists in space even when there is no conducting wire present.

The electric field need not be conservative

In electrostatics,

$$\nabla \times \mathbf{E} = 0,$$

so the electric field can be written as

$$\mathbf{E} = -\nabla V.$$

But when magnetic flux varies with time,

$$\nabla \times \mathbf{E} \neq 0.$$

The electric field then has circulation and cannot generally be represented everywhere by a single scalar electrostatic potential alone.

This is a major conceptual transition from electrostatics to electrodynamics.

Orientation and the right-hand rule

The direction assigned to $d\mathbf{S}$ and the positive traversal direction around C are linked by the right-hand rule. Reverse the surface normal and the positive contour direction must reverse too. The law remains the same because both signed quantities change consistently.

RF interpretation

In an RF system, fields vary sinusoidally. A changing magnetic field continuously creates a circulating electric field. Later, in phasor form, the time derivative becomes multiplication by $j\omega$ for the $e^{j\omega t}$ convention:

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B}.$$

Document 8 will develop that frequency-domain form in detail.

Core idea: Faraday's law says that electric circulation can be produced without electric charge. A changing magnetic field itself is sufficient.

5. Ampère-Maxwell law and displacement current

The fourth Maxwell equation is

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t}.$$

Its integral form is

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \iint_S \mathbf{J}_f \cdot d\mathbf{S} + \frac{d}{dt} \iint_S \mathbf{D} \cdot d\mathbf{S}.$$

The first term on the right represents **conduction current** through the surface. The second is the **displacement-current term** introduced by Maxwell.

Why Ampère's original law was incomplete

For steady currents,

$$\nabla \times \mathbf{H} = \mathbf{J}$$

works well. The difficulty appears for time-varying charge. Consider a capacitor being charged through wires. Conduction current flows in the wires, but no conduction current crosses the dielectric gap between the plates.

If we apply the circulation law to a loop around one wire, two different surfaces can be stretched across that same loop. One surface cuts the wire and encloses conduction current. Another bows between the capacitor plates and cuts no conduction current. A physical law cannot depend on which imaginary surface we choose.

Maxwell resolved this by adding

$$\frac{\partial \mathbf{D}}{\partial t}.$$

Between the capacitor plates, the electric flux changes as charge accumulates. The displacement-current contribution exactly restores consistency.

Define a displacement-current density

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}.$$

Then the curl law can be remembered as

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \mathbf{J}_d.$$

Why this term is profound

The displacement-current term is not merely bookkeeping. In a source-free dielectric region, $\mathbf{J}_f = 0$, yet a time-varying electric field can still produce magnetic circulation:

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}.$$

Together with Faraday's law,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

this gives the mutual electric-magnetic coupling needed for electromagnetic-wave propagation.

For a linear material, conduction current is $\mathbf{J}_c = \sigma \mathbf{E}$, while displacement current is $\mathbf{J}_d = \partial \mathbf{D} / \partial t$. In sinusoidal steady state, Document 8 will show that these become algebraic quantities involving σ and $j\omega\epsilon$.

Teaching point: displacement current makes Ampère's law consistent with changing charge and is essential to the existence of electromagnetic waves.

6. Charge conservation and the continuity equation

Maxwell's equations contain charge conservation within them. The local conservation law is the **continuity equation**:

$$\nabla \cdot \mathbf{J}_f = -\frac{\partial \rho_f}{\partial t}.$$

This says that if net current flows outward from a tiny volume, the charge stored in that volume must decrease. Conversely, if more current flows in than out, charge accumulates.

We can derive the equation directly from Ampère-Maxwell law:

$$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}.$$

Take the divergence of both sides:

$$\nabla \cdot (\nabla \times \mathbf{H}) = \nabla \cdot \mathbf{J}_f + \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D}).$$

The divergence of a curl is identically zero:

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0.$$

Using Gauss's law,

$$\nabla \cdot \mathbf{D} = \rho_f,$$

we obtain

$$0 = \nabla \cdot \mathbf{J}_f + \frac{\partial \rho_f}{\partial t},$$

or

$$\boxed{\nabla \cdot \mathbf{J}_f = -\frac{\partial \rho_f}{\partial t}}.$$

Integral form

Integrating over a volume V and using the divergence theorem gives

$$\oint_S \mathbf{J}_f \cdot d\mathbf{S} = -\frac{d}{dt} \iiint_V \rho_f dV.$$

Since the volume integral is the enclosed charge Q_f ,

$$\boxed{\oint_S \mathbf{J}_f \cdot d\mathbf{S} = -\frac{dQ_f}{dt}}.$$

The left side is net current leaving the volume. If that is positive, enclosed charge falls.

Why this matters in EM modelling

Charge conservation is not optional. A field/current solution that violates the continuity equation is internally inconsistent. Numerical solvers therefore need source representations, meshes and matrix equations that respect charge continuity sufficiently well.

For example, in a Method of Moments current formulation, the basis functions should represent current flow in a way compatible with charge accumulation. In FDTD and FEM, discrete versions of divergence and curl identities help maintain consistency between field updates and charge.

A practical interpretation

Imagine a short conducting segment carrying a time-varying current. If the current entering one end differs from the current leaving the other, charge must accumulate somewhere on the segment. That accumulated charge produces electric field. Current distribution and charge distribution are therefore not independent quantities.

Core idea: the continuity equation is conservation of electric charge written locally. It is embedded in Maxwell's equations, not added as an unrelated law.

7. Integral and differential forms: why both matter

The same Maxwell law can often be expressed either as a statement about a **finite region** or as a statement about **every point**. The bridge between these views is provided by the divergence theorem and Stokes' theorem.

For the divergence equations, the divergence theorem says

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \iiint_V \nabla \cdot \mathbf{A} dV.$$

Applying it to electric flux gives

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \iiint_V \nabla \cdot \mathbf{D} dV = \iiint_V \rho_f dV.$$

Thus the global statement about total enclosed charge and the local statement $\nabla \cdot \mathbf{D} = \rho_f$ are two views of the same law.

For the curl equations, Stokes' theorem says

$$\oint_C \mathbf{A} \cdot d\mathbf{l} = \iint_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S}.$$

Apply this to Faraday's law:

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = \iint_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S}.$$

But Faraday's integral law also gives

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint_S \mathbf{B} \cdot d\mathbf{S}.$$

For a fixed surface we can move the time derivative inside the integral:

$$\iint_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = - \iint_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}.$$

Hence

$$\iint_S \left(\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} \right) \cdot d\mathbf{S} = 0.$$

Because the surface is arbitrary,

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}}.$$

When should you use each form?

The **integral form** is often natural when there is strong symmetry, when dealing with total flux/current, or when deriving boundary conditions around a small loop or pillbox.

The **differential form** is natural when deriving wave equations, solving PDEs, or describing the local field inside a material.

Computational methods reflect this distinction. FDTD and many FEM formulations are closely tied to local differential operators. MoM typically starts from integral equations in which Green's functions have already incorporated part of the differential physics.

Teaching point: integral Maxwell equations are not approximations to the differential equations. For sufficiently regular fields, they are mathematically equivalent descriptions of the same physics.

8. Boundary conditions from Maxwell's equations

Boundary conditions tell us how fields behave as we cross an interface between two materials. They are not arbitrary rules; they follow directly from Maxwell's integral equations by applying them to an infinitesimally small loop or pillbox around the boundary.

Let $\hat{\mathbf{n}}$ point from medium 1 into medium 2.

Normal electric-flux condition

Apply Gauss's law to a very thin pillbox straddling the boundary. As its height tends to zero, only the two flat faces contribute appreciably. The result is

$$\hat{\mathbf{n}} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \rho_s$$

where ρ_s is free surface-charge density.

If there is no free surface charge,

$$\hat{\mathbf{n}} \cdot \mathbf{D}_2 = \hat{\mathbf{n}} \cdot \mathbf{D}_1.$$

Normal magnetic-flux condition

Applying $\nabla \cdot \mathbf{B} = 0$ to the same pillbox gives

$$\hat{\mathbf{n}} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0.$$

The normal component of $\mathbf{B}$ is continuous because there is no magnetic surface charge.

Tangential electric-field condition

Apply Faraday's law to an infinitesimal rectangular loop crossing the interface. For finite fields, the magnetic-flux term tends to zero as the loop area shrinks, leaving

$$\hat{\mathbf{n}} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0.$$

Thus the tangential component of $\mathbf{E}$ is continuous across an ordinary interface.

Tangential magnetic-field condition

Applying Ampère-Maxwell law to the same type of loop gives

$$\hat{\mathbf{n}} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{J}_s$$

where $\mathbf{J}_s$ is free surface-current density.

If no surface current exists, the tangential component of $\mathbf{H}$ is continuous.

These four conditions are fundamental to reflection/refraction problems, transmission lines, waveguides, antennas, dielectric interfaces and numerical EM.

Perfect electric conductor

Inside an ideal PEC, the time-varying electric field is zero. At its surface this implies

$$\mathbf{E}_t = 0.$$

The normal electric flux may terminate on surface charge, and tangential magnetic field may be supported by surface current. This is why PEC boundary conditions are so important in MoM and waveguide problems.

Core idea: every familiar EM boundary condition can be traced back to one of the four Maxwell equations applied to a vanishingly small geometric region.

9. How Maxwell's equations generate electromagnetic waves

The complete wave-equation derivation belongs to Document 9, but it is important to see the logic now.

Consider a homogeneous, source-free, linear medium:

$$\rho_f = 0, \quad \mathbf{J}_f = 0,$$

with

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}.$$

Maxwell's curl equations become

$$\begin{aligned} \nabla \times \mathbf{E} &= -\mu \frac{\partial \mathbf{H}}{\partial t}, \\ \nabla \times \mathbf{H} &= \epsilon \frac{\partial \mathbf{E}}{\partial t}. \end{aligned}$$

The first says a changing magnetic field is associated with electric circulation. The second says a changing electric field is associated with magnetic circulation. Each field therefore drives the spatial variation of the other.

Take the curl of Faraday's law:

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu \frac{\partial}{\partial t} (\nabla \times \mathbf{H}).$$

Substitute Ampère-Maxwell law:

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}.$$

Using the vector identity

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E},$$

and noting that in this homogeneous source-free region

$$\nabla \cdot \mathbf{E} = 0,$$

we obtain

$$\nabla^2 \mathbf{E} = \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}.$$

This has the standard wave-equation form, giving wave speed

$$v = \frac{1}{\sqrt{\mu \epsilon}}.$$

In vacuum,

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

The remarkable point is conceptual: **electromagnetic waves are a consequence of Maxwell's field laws and material relations.** We do not need to add a separate law saying that radio waves exist.

This connection is why the displacement-current term in Ampère-Maxwell law is indispensable. Without it, the symmetrical electric-magnetic coupling required for free-space waves would be lost.

Teaching point: a propagating EM wave is not an electric wave plus an unrelated magnetic wave. It is one coupled electromagnetic field solution in which $\mathbf{E}$ and $\mathbf{H}$ continually satisfy Maxwell's equations together.

10. Synthesis, solver perspective and degree-level checkpoint

A useful way to memorise Maxwell's equations is by what they **do**, not merely by their symbols.

Law	Differential form	Physical statement
Electric Gauss	$\nabla \cdot \mathbf{D} = \rho_f$	charge is a source/sink of electric flux
Magnetic Gauss	$\nabla \cdot \mathbf{B} = 0$	magnetic flux has no isolated source/sink
Faraday	$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$	changing magnetic flux creates electric circulation
Ampère-Maxwell	$\nabla \times \mathbf{H} = \mathbf{J}_f + \partial \mathbf{D} / \partial t$	current and changing electric flux create magnetic circulation

The field equations must be used together with material relations such as

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}, \quad \mathbf{J} = \sigma \mathbf{E}.$$

What an EM solver is really solving

Different numerical methods rearrange Maxwell's equations in different ways.

FDTD discretises the curl equations directly in space and time and updates electric and magnetic fields alternately.

FEM typically converts a differential Maxwell equation into a weak integral form over mesh elements and solves the resulting sparse matrix system.

MoM often starts from an integral equation in which a Green's function already represents the response of Maxwell's differential operator; the unknown source/current distribution is expanded in basis functions and solved from a dense matrix system.

So although the software may display matrices rather than curls and fluxes, the underlying physics remains Maxwell.

Degree-level checkpoint

You should now be able to answer the following without relying on memorised slogans.

1. State all four Maxwell equations in differential form and explain every term and unit.
2. State all four in integral form and identify the line, surface or volume involved.
3. Derive $\nabla \cdot \mathbf{D} = \rho_f$ from integral Gauss's law using the divergence theorem.
4. Derive Faraday's differential law from its integral form using Stokes' theorem.
5. Explain physically why $\nabla \cdot \mathbf{D} = 0$ does not imply $\mathbf{D} = 0$.
6. Explain why the displacement-current term is required for a charging capacitor.
7. Derive the continuity equation from Ampère-Maxwell law and Gauss's law.
8. Explain the meaning of $\nabla \cdot \mathbf{B} = 0$ without saying merely "there are no monopoles".
9. Derive the four basic interface boundary conditions from Maxwell's equations.
10. Explain, in words and equations, how the two curl laws permit electromagnetic-wave propagation.

Teach-back challenge

A good teaching test is to explain Maxwell's equations in three passes:

Pass 1 — physical language: sources, flux, circulation and induction.

Pass 2 — integral equations: what happens over loops, surfaces and enclosed volumes.

Pass 3 — differential equations: what happens locally at each point.

If you can move comfortably between those three descriptions, and can explain why the mathematical operators match the physical ideas, you have crossed the most important conceptual threshold in undergraduate electromagnetics.

Next document: sinusoidal fields and the phasor-domain Maxwell equations. This will show how time differentiation becomes algebraic multiplication by $j\omega$, which is the form used throughout RF, microwave and frequency-domain EM analysis.